

# Open String Fields as Matrices

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## Abstract

We show that the action expanded around Erler-Maccaferri's  $N$  D-brane solution describes the  $N + 1$  D-brane system where one D-brane disappears due to tachyon condensation. String fields on multi-branes can be regarded as block matrices of a string field on a single D-brane in the same way as matrix theories.

# 1 Introduction

Open string field theory has the possibility of revealing non-perturbative aspects of string theory. Recently, Erler and Maccaferri have proposed a method to construct classical solutions, which are expected to describe any open string background [1]. This indeed implies that open string field theory is able to give a unified description of various D-branes regarded as non-perturbative objects of string theory.

Multi-brane solutions in Ref. [1] provide a correct vacuum energy and gauge invariant observables. Accordingly, in order to prove whether the theory describes a multi-brane system, it is necessary to clarify open and closed string spectra in the background of the solution. However, it is difficult to give a definite answer to this problem, because there are some subtleties concerning BRST cohomology in the background [1].

There is another question related to the degree of freedom of string fields in the background. We have one string field in the theory on a single D-brane. However, in the case that the multi-brane solution provides the background of the  $N$  D-branes, the number of string fields increases to  $N^2$  around the solution. Intuitively,  $N^2$  fluctuation fields in the multi-brane background seem to be introduced as redundant degrees of freedom. Here, it is natural to ask how to generate  $N^2$  string fields or Chan-Paton factors from one string field.

On the other hand, it is well-known that matrix theories are able to describe various D-branes [2, 3]. In matrix theories, D-branes are created by classical solutions as block diagonal matrices. After expanding a matrix around the solution, block matrices can be understood as representing open strings connecting each D-brane. Here, it should be noted that there are similarities between the matrix and the open string field: the matrix is deeply tied to the open string degree of freedom and an open string field is interpreted as a matrix in which the left and right indices correspond to the left and right half-strings [4]. Then, it seems plausible that  $N^2$  string fields on  $N$  D-branes are embedded like block matrices in a string field on a D-brane.

The purpose of this paper is to clarify the origin of the  $N^2$  string fields in the background of an  $N$  D-brane solution. We will show that the theory expanded around the solution is regarded as an open string field theory on  $N + 1$  D-branes, but in which a D-brane vanishes as a result of tachyon condensation. Then, the  $N^2$  string fields will be given as block matrices in a string field as an infinite-dimensional matrix. Consequently, we can expect that the  $N$  D-brane solution correctly reproduces the open and closed string spectra in the  $N$  D-brane background.

The paper is organized as follows. In Sect. 2, after a brief explanation of multi-brane solutions by Erler-Maccaferri [1], we will introduce projection operators acting on a space of string fields. Then, we will analyze a string field theory expanded around the  $N$  D-brane solution in terms of the projectors. In Sect. 3, we will give concluding remarks.

## 2 Open string field theory around multi-brane solutions

### 2.1 Erler-Maccaferri's solution for $N$ D-branes

The action of bosonic cubic open string field theory is

$$S[\Psi; Q_B] = -\frac{1}{g^2} \int \left( \frac{1}{2} \Psi Q_B \Psi + \frac{1}{3} \Psi^3 \right). \quad (2.1)$$

From the action, the equation of motion is given by

$$Q_B \Psi + \Psi^2 = 0. \quad (2.2)$$

To construct multi-brane solutions for  $N$  D-branes, Erler and Maccaferri introduced  $N$  pairs of regularized boundary-conditions-changing operators,  $\Sigma_a$  and  $\bar{\Sigma}_a$  ( $a = 1, \dots, N$ ) [1].<sup>1</sup> These operators satisfy

$$\bar{\Sigma}_a \Sigma_b = \delta_{ab}, \quad (2.4)$$

and

$$Q_T \Sigma_a = Q_T \bar{\Sigma}_a = 0, \quad (2.5)$$

where  $Q_T$  is a modified BRST operator on the tachyon vacuum. From (2.5), we find that

$$Q_B \Sigma_a = \Sigma_a \psi_T - \psi_T \Sigma_a, \quad Q_B \bar{\Sigma}_a = \bar{\Sigma}_a \psi_T - \psi_T \bar{\Sigma}_a, \quad (2.6)$$

where  $\psi_T$  denotes the tachyon vacuum solution of (2.2). Here, we only assume that  $\Sigma_a$  and  $\bar{\Sigma}_a$  satisfy Eqs. (2.4) and (2.5) (or equivalently (2.6)) for a tachyon vacuum solution  $\psi_T$ , regardless of wedge-based[5, 6] or identity-based[7, 8] solutions.

Using  $\psi_T$ ,  $\Sigma_a$  and  $\bar{\Sigma}_a$ , Erler-Maccaferri provided a multi-brane solution as[1]

$$\Psi_0 = \psi_T - \sum_{a=1}^N \Sigma_a \psi_T \bar{\Sigma}_a. \quad (2.7)$$

We can calculate the action for  $\Psi_0$  with the help of (2.4) and (2.5):

$$S[\Psi_0; Q_B] = -(N-1)S[\psi_T; Q_B]. \quad (2.8)$$

Then, the solution  $\Psi_0$  provides a correct vacuum energy for  $N$  D-branes. Expanding the string field around the solution as  $\Psi = \Psi_0 + \psi$ , we can obtain the action for the fluctuation  $\psi$ :

$$S[\Psi; Q_B] = S[\Psi_0; Q_B] + S[\psi; Q_{\Psi_0}], \quad (2.9)$$

where the operator  $Q_{\Psi_0}$  denotes the shifted BRST operator by the solution  $\Psi_0$ .

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<sup>1</sup> $\Sigma_a$  and  $\bar{\Sigma}_a$  are constructed by boundary-condition-changing (bcc) operators,  $\sigma_a$  and  $\bar{\sigma}_a$ , satisfying the operator product expansion:  $\bar{\sigma}_a(z')\sigma_b(z) \rightarrow \delta_{ab} (z' \rightarrow z)$ . In the Minkowski background, a zero momentum condition for the bcc operators is not necessarily required. So, the simplest bcc operators are given as

$$\sigma_a(z) = e^{ik_a \cdot X(z)}, \quad \bar{\sigma}_a(z) = e^{-ik_a \cdot X(z)}, \quad (2.3)$$

where  $k_a^\mu$  satisfy  $k_a^2 = 0$  and  $k_a \cdot k_b < 0$  ( $a \neq b$ ). For example, we can take  $k_a^\mu = (a, 1, \sqrt{a^2 - 1}, 0, \dots, 0)$ .

## 2.2 Projectors

To clarify the physical interpretation of  $S[\psi; Q_{\Psi_0}]$ , we introduce  $N$  projection states as follows:

$$P_a = \Sigma_a \bar{\Sigma}_a \quad (a = 1, \dots, N), \quad (2.10)$$

where the same indices  $a$  are not summed. Here we have to notice that, as pointed out in Ref. [1],  $\bar{\Sigma}_a$  should be multiplied to  $\Sigma_a$  from the left and so these projectors should be dealt with carefully. More precisely, we define the projections for arbitrary string fields  $A$  and  $B$  as follows:

$$AP_a B = (A \Sigma_a)(\bar{\Sigma}_a B). \quad (2.11)$$

From (2.4) and (2.11), we can easily find that

$$P_a P_b A = \Sigma_a (\bar{\Sigma}_a P_b A) = \Sigma_a ((\bar{\Sigma}_a \Sigma_b)(\bar{\Sigma}_b A)) = \delta_{ab} P_a A. \quad (2.12)$$

This is a sufficient definition of the projectors for later calculation. But it suggests that we need to insert some infinitesimal worldsheet to separate  $\Sigma_a$  and  $\bar{\Sigma}_a$ . We will discuss this point further in the last section.

In addition to  $P_a$ , we define the 0th projection as a complementary projector:

$$P_0 = 1 - \sum_{a=1}^N P_a, \quad (2.13)$$

where 1 denotes the identity string field. By definition, these  $N + 1$  projections satisfy

$$\sum_{\alpha=0}^N P_\alpha = 1, \quad (2.14)$$

where the Greek indices are used for values  $0, 1, \dots, N$ . From (2.5), it follows that  $Q_T P_\alpha = 0$  and then we have

$$Q_B P_\alpha = P_\alpha \psi_T - \psi_T P_\alpha. \quad (2.15)$$

Moreover, we can find some relations among  $P_\alpha$ ,  $\Sigma_\alpha$  and  $\bar{\Sigma}_\alpha$ :

$$P_a \Sigma_b = \Sigma_a \delta_{ab}, \quad \bar{\Sigma}_a P_b = \bar{\Sigma}_a \delta_{ab}, \quad P_0 \Sigma_b = 0, \quad \bar{\Sigma}_a P_0 = 0. \quad (2.16)$$

With the help of these projectors, the string field  $\Psi$  can be partitioned into  $(N+1) \times (N+1)$  blocks:

$$\Psi = \sum_{\alpha=0}^N \sum_{\beta=0}^N P_\alpha \Psi P_\beta = \begin{pmatrix} \Psi_{00} & \Psi_{01} & \cdots & \Psi_{0N} \\ \Psi_{10} & \Psi_{11} & \cdots & \Psi_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_{N0} & \Psi_{N1} & \cdots & \Psi_{NN} \end{pmatrix}, \quad (2.17)$$

where  $\Psi_{\alpha\beta}$  is defined as the  $(\alpha, \beta)$  sector of  $\Psi$ , i.e.,  $\Psi_{\alpha\beta} \equiv P_\alpha \Psi P_\beta$ .

According to Ref. [1], the second term in (2.7) is a solution to the equation of motion at the tachyon vacuum. From (2.16), the second term is represented as

$$-\sum_{a=1}^N \Sigma_a \psi_T \bar{\Sigma}_a = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & -\Sigma_1 \psi_T \bar{\Sigma}_1 & 0 & \cdots & 0 \\ 0 & 0 & -\Sigma_2 \psi_T \bar{\Sigma}_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -\Sigma_N \psi_T \bar{\Sigma}_N \end{pmatrix}. \quad (2.18)$$

Accordingly, it turns out that the  $N$  D-brane solution at the tachyon vacuum is given as a block diagonal matrix. This is a similar result to the case of matrix theories [2, 3].

### 2.3 Background described by the solution

Now, we consider the fluctuation  $\psi$  around the  $N$  D-brane solution. According to the previous subsection,  $\psi$  can be written by matrix representation:

$$\psi = \sum_{\alpha=0}^N \sum_{\beta=0}^N \tilde{\phi}_{\alpha\beta}, \quad (2.19)$$

where  $\tilde{\phi}_{\alpha\beta} = P_\alpha \psi P_\beta$ .  $\tilde{\phi}_{\alpha\beta}$  represents a block matrix of  $\psi$  with infinite dimension.

Here, we consider change of variables of  $\tilde{\phi}_{\alpha\beta}$ .  $\tilde{\phi}_{ab}$  can be rewritten as

$$\tilde{\phi}_{ab} = P_a \tilde{\phi}_{ab} P_b = \Sigma_a (\bar{\Sigma}_a \tilde{\phi}_{ab} \Sigma_b) \bar{\Sigma}_b. \quad (2.20)$$

So, we can change the variables from  $\tilde{\phi}_{ab}$  to  $\phi_{ab} = \bar{\Sigma}_a \tilde{\phi}_{ab} \Sigma_b$ . Similarly, writing  $\tilde{\phi}_{0a} = \chi_a \bar{\Sigma}_a$ ,  $\tilde{\phi}_{a0} = \Sigma_a \bar{\chi}_a$ , the fluctuation  $\psi$  is represented as

$$\begin{aligned} \psi &= \chi + \sum_{a=1}^N \chi_a \bar{\Sigma}_a + \sum_{a=1}^N \Sigma_a \bar{\chi}_a + \sum_{a=1}^N \sum_{b=1}^N \Sigma_a \phi_{ab} \bar{\Sigma}_b \\ &= \begin{pmatrix} \chi & \chi_b \bar{\Sigma}_b \\ \Sigma_a \bar{\chi}_a & \Sigma_a \phi_{ab} \bar{\Sigma}_b \end{pmatrix}, \end{aligned} \quad (2.21)$$

where we rewrite  $\tilde{\phi}_{00}$  as  $\chi$ .

Similar to the equation  $Q_{\Psi_0}(\Sigma_a A \bar{\Sigma}_b) = \Sigma_a (Q_B A) \bar{\Sigma}_b$  given in Ref. [1], by using (2.6) and (2.15), we have

$$Q_{\Psi_0}(P_0 A P_0) = P_0 (Q_T A) P_0, \quad (2.22)$$

$$Q_{\Psi_0}(P_0 A \bar{\Sigma}_a) = P_0 (Q_{T0} A) \bar{\Sigma}_a, \quad (2.23)$$

$$Q_{\Psi_0}(\Sigma_a A P_0) = \Sigma_a (Q_{0T} A) P_0, \quad (2.24)$$

where the operator  $Q_{\psi_1\psi_2}$  is defined as  $Q_{\psi_1\psi_2}A = Q_B A + \psi_1 A - (-1)^{|A|} A \psi_2$  for two classical solutions  $\psi_1$  and  $\psi_2$  [1], and then  $Q_{T0} \equiv Q_{\psi_T 0}$  and  $Q_{0T} \equiv Q_{0\psi_T}$ . Using these relations, we can obtain a matrix representation of  $Q_{\Psi_0}\psi$ :

$$Q_{\Psi_0}\psi = \begin{pmatrix} P_0(Q_T\chi)P_0 & P_0(Q_{T0}\chi_b)\bar{\Sigma}_b \\ \Sigma_a(Q_{0T}\bar{\chi}_a)P_0 & \Sigma_a(Q_B\phi_{ab})\bar{\Sigma}_b \end{pmatrix}. \quad (2.25)$$

Consequently, the action expanded around  $\Psi_0$  can be rewritten as

$$S[\psi; Q_{\Psi_0}] = S[\phi_{ab}; Q_B] + S'[\chi, \chi_a, \bar{\chi}_a, \phi_{ab}], \quad (2.26)$$

where each action is given by

$$\begin{aligned} S[\phi_{ab}; Q_B] &= -\frac{1}{g^2} \int \left( \frac{1}{2} \sum_{a=1}^N \sum_{b=1}^N \phi_{ba} Q_B \phi_{ab} + \frac{1}{3} \sum_{a=1}^N \sum_{b=1}^N \sum_{c=1}^N \phi_{ab} \phi_{bc} \phi_{ca} \right) \\ &= -\frac{1}{g^2} \int \text{tr} \left( \frac{1}{2} \phi Q_B \phi + \frac{1}{3} \phi^3 \right), \end{aligned} \quad (2.27)$$

and

$$\begin{aligned} S'[\chi, \chi_a, \bar{\chi}_a, \phi_{ab}] &= -\frac{1}{g^2} \int \left( \frac{1}{2} \chi Q_T \chi + \sum_{a=1}^N \bar{\chi}_a Q_{T0} \chi_a + \frac{1}{3} \chi^3 \right. \\ &\quad \left. + \sum_{a=1}^N \bar{\chi}_a \chi \chi_a + \sum_{a=1}^N \sum_{b=1}^N \chi_a \phi_{ab} \bar{\chi}_b \right). \end{aligned} \quad (2.28)$$

In (2.27),  $\phi$  represents a matrix ( $\phi_{ab}$ ) and the trace denotes the sum of the diagonal elements with indices  $a, b$ . Obviously, (2.27) represents the action for  $N$  D-branes; namely,  $\phi_{ab}$  is a string field of an open string attached on the  $a$ th and  $b$ th D-branes. Moreover, in the action (2.28),  $\chi$  is a string field on a D-brane with tachyon condensation, and  $\chi_a$  and  $\bar{\chi}_a$  represent string fields of an open string attaching on a D-brane with tachyon condensation and on one of the  $N$  D-branes, on which  $\phi_{ab}$  also attach. Accordingly, the actions (2.27) and (2.28) describe the theory for  $N + 1$  D-branes in which a D-brane vanishes due to tachyon condensation. This system should be physically equivalent to the  $N$  D-brane system because  $Q_T$  and  $Q_{T0}$  have trivial cohomology<sup>2</sup> and therefore this result is consistent with the expectation that the solution (2.7) is regarded as an  $N$  D-brane solution.

Let us consider an on-shell closed string coupling to an open string field. In the complex plane, a closed string vertex operator is given by  $\mathcal{V}(z, \bar{z}) = c(z)c(\bar{z})V_{\text{matt}}(z, \bar{z})$ , where  $V_{\text{matt}}$  is a vertex operator with the conformal dimension  $(1, 1)$  in the matter sector. We can give a BRST invariant state using  $\mathcal{V}$  as

$$V = \mathcal{V}(i, -i)I, \quad (2.29)$$

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<sup>2</sup> In Ref. [9], it is shown that a homotopy operator exists for  $Q_T$ ,  $Q_{T0}$ , and  $Q_{0T}$  if a homotopy state is given for  $Q_T$ . For the identity-based tachyon vacuum solution[10],  $Q_{T0}$  and  $Q_{0T}$  also have vanishing cohomology, as does  $Q_T$  [8], since a homotopy state can be constructed for the solution.

where the point  $z = i$  corresponds to the midpoint of an open string. Since the vertex is inserted at the midpoint, the state  $V$  commutes with any string field  $A$ :  $VA = AV$ . For the open string field  $\Psi$ , an interaction term with the closed string vertex is given as a gauge invariant overlap[11]:

$$O_V(\Psi) = \int V\Psi. \quad (2.30)$$

In the background of the  $N$  D-brane solution, using (2.4) and (2.16), we can easily find couplings of the fluctuation fields to the closed string as

$$O_V(\psi) = O_V(\chi) + \sum_{a=1}^N O_V(\phi_{aa}). \quad (2.31)$$

This correctly provides a closed string interaction to open strings on the  $N + 1$  D-branes.

Next, we consider the correspondence between gauge symmetries in the original action (2.1) and the expanded action (2.26). The original gauge transformation is given by

$$\delta_\Lambda \Psi = Q_B \Lambda + \Psi \Lambda - \Lambda \Psi. \quad (2.32)$$

Since  $\Psi = \Psi_0 + \psi$ , the gauge transformation for  $\psi$  is given by

$$\delta_\Lambda \psi = Q_{\Psi_0} \Lambda + \psi \Lambda - \Lambda \psi, \quad (2.33)$$

where we note that  $\Lambda$  is the same parameter as in (2.32). Here, we decompose  $\Lambda$  into  $\tilde{\Lambda}_{\alpha\beta} = P_\alpha \Lambda P_\beta$  by the projectors. Then, changing variables as

$$\tilde{\Lambda}_{ab} = \Sigma_a \Lambda_{ab} \bar{\Sigma}_b, \quad \tilde{\Lambda}_{0a} = \lambda_a \bar{\Sigma}_a, \quad \tilde{\Lambda}_{a0} = \Sigma_a \bar{\lambda}_a, \quad (2.34)$$

and writing  $\tilde{\Lambda}_{00} = \lambda$ , we find that

$$\delta_\Lambda \psi = \sum_{a=1}^N \sum_{b=1}^N \Sigma_a (\delta_\Lambda \phi_{ab}) \bar{\Sigma}_b + \sum_{a=1}^N P_0 (\delta_\Lambda \chi_a) \bar{\Sigma}_a + \sum_{a=1}^N \Sigma_a (\delta_\Lambda \bar{\chi}_a) P_0 + P_0 (\delta_\Lambda \chi) P_0, \quad (2.35)$$

where the gauge transformations for the components are given as

$$\begin{aligned} \delta_\Lambda \phi_{ab} &= Q_B \Lambda_{ab} + \phi_{ac} \Lambda_{cb} - \Lambda_{ac} \phi_{cb} + \bar{\chi}_a \lambda_b - \bar{\lambda}_a \chi_b, \\ \delta_\Lambda \chi_a &= P_0 (Q_{T0} \lambda_a) + \chi \lambda_a + \chi_b \Lambda_{ba} - \lambda \chi_a - \lambda_b \phi_{ba}, \\ \delta_\Lambda \bar{\chi}_a &= (Q_{0T} \bar{\lambda}_a) P_0 + \bar{\chi}_a \lambda + \phi_{ab} \bar{\lambda}_b - \bar{\lambda}_a \chi - \Lambda_{ab} \bar{\chi}_b, \\ \delta_\Lambda \chi &= P_0 (Q_T \lambda) P_0 + \chi \lambda + \chi_a \bar{\lambda}_a - \lambda \chi - \lambda_a \bar{\chi}_a. \end{aligned} \quad (2.36)$$

### 3 Concluding remarks

We have shown that the theory expanded around the  $N$  D-brane solution given by Erler-Maccaferri describes an  $N + 1$  D-brane system with a vanishing D-brane due to the tachyon

condensation. By projectors made of regularized bcc operators, an open string field in the original theory is divided into multi-string fields with matrix indices. Then, these indices can be regarded as Chan-Paton factors in the  $N$  D-brane background. We have found that  $N^2$  string fields on  $N$  D-branes are embedded in a string field as block matrices. Similarly, gauge transformation parameters in the expanded theory are represented as block elements of a gauge parameter string field in the original theory.

From the matrix representation (2.19), the string fields  $\tilde{\phi}_{\alpha\beta}$  are mutually independent variables and then the degrees of freedom of  $\tilde{\phi}_{\alpha\beta}$  are equivalent to those of the string field  $\psi$ . Then, it is natural to expect that the path integral measure of the fluctuation  $\psi$  is given by the product of measures of  $\tilde{\phi}_{\alpha\beta}$ . As seen in the previous section, we can rewrite  $\tilde{\phi}_{\alpha\beta}$  as  $\phi_{ab}$ ,  $\chi$ ,  $\chi_a$ , and  $\bar{\chi}_a$  by linear transformations. Therefore, the measure of  $\psi$  is expressed by the measures of the string fields on the  $N + 1$  D-branes:

$$\mathcal{D}\psi = \prod_{a=1}^N \prod_{b=1}^N \mathcal{D}\phi_{ab} \mathcal{D}\chi \prod_{a=1}^N \mathcal{D}\chi_a \prod_{a=1}^N \mathcal{D}\bar{\chi}_a. \quad (3.1)$$

Hence, the matrix interpretation of open string fields ensures that the quantum measure for the  $N$  D-brane system is correctly derived from the classical solution in the string field theory.

Finally, we should comment on the multiplicative ordering of  $\Sigma_a$  and  $\bar{\Sigma}_a$  in the projectors. As in (2.11), we have defined the projectors such that  $\Sigma_a$  does not operate on  $\bar{\Sigma}_a$ , because bcc operators break associativity, as discussed in Ref. [1]. To get a more definite result, we should separate these states by some worldsheet. This is a similar approach to that adopted in Ref. [12] to remedy the problem due to another nonassociativity. Accordingly, we need to regularize  $P_a$  by inserting some worldsheet between  $\Sigma_a$  and  $\bar{\Sigma}_a$ . In the case that  $\psi_T$  is given by the Erler-Schnabl solution, one possible choice for regularization is

$$P_a = \Sigma_a Q_T \left( \frac{B}{1+K} e^{-\epsilon K} \right) \bar{\Sigma}_a, \quad (3.2)$$

where  $\epsilon$  is a positive infinitesimal parameter. It is noted that  $B/(1+K)$  is a homotopy operator for  $Q_T$  and this construction is parallel to that of the regularized bcc operators from  $\sigma$  and  $\bar{\sigma}$  [1]. It can easily be seen that  $P_a P_b = \delta_{ab} P_a$  and  $Q_T P_a = 0$ .

In this regularization, the limit  $\epsilon \rightarrow 0$  should be taken after calculating the correlation functions related to trace (or integration) of string fields. It should never be done in string fields; e.g., the state  $P_a A$  keeps the parameter  $\epsilon$  until correlation functions are calculated. Evidently, the state with the regularization parameter is regarded as a kind of distribution as in Ref. [13] and indeed it is outside the usual Fock space like the phantom term in Schnabl's tachyon vacuum solution [5]. We hope that, in terms of the projectors, it will be possible to obtain a deeper understanding of a space of string fields, in particular, the topology in the space beyond the single Fock space in string field theories [14].

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## References

- [1] T. Erler and C. Maccaferri, “String Field Theory Solution for Any Open String Background,” arXiv:1406.3021 [hep-th].
- [2] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M theory as a matrix model: A Conjecture,” Phys. Rev. D **55**, 5112 (1997) [hep-th/9610043].
- [3] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, “A Large N reduced model as superstring,” Nucl. Phys. B **498**, 467 (1997) [hep-th/9612115].
- [4] E. Witten, “Noncommutative Geometry and String Field Theory,” Nucl. Phys. B **268**, 253 (1986).
- [5] M. Schnabl, “Analytic solution for tachyon condensation in open string field theory,” Adv. Theor. Math. Phys. **10**, 433 (2006) [hep-th/0511286].
- [6] T. Erler and M. Schnabl, “A Simple Analytic Solution for Tachyon Condensation,” JHEP **0910**, 066 (2009) [arXiv:0906.0979 [hep-th]].
- [7] T. Takahashi and S. Tanimoto, “Marginal and scalar solutions in cubic open string field theory,” JHEP **0203**, 033 (2002) [hep-th/0202133].
- [8] I. Kishimoto and T. Takahashi, “Open string field theory around universal solutions,” Prog. Theor. Phys. **108**, 591 (2002) [hep-th/0205275].
- [9] I. Ellwood and M. Schnabl, “Proof of vanishing cohomology at the tachyon vacuum,” JHEP **0702**, 096 (2007) [hep-th/0606142].
- [10] S. Inatomi, I. Kishimoto and T. Takahashi, “Homotopy Operators and One-Loop Vacuum Energy at the Tachyon Vacuum,” Prog. Theor. Phys. **126**, 1077 (2011) [arXiv:1106.5314 [hep-th]].
- [11] B. Zwiebach, “Interpolating string field theories,” Mod. Phys. Lett. A **7**, 1079 (1992) [hep-th/9202015].
- [12] N. Ishibashi, “Comments on Takahashi-Tanimoto’s scalar solution,” arXiv:1408.6319 [hep-th].
- [13] L. Bonora and S. Giaccari, “Generalized states in SFT,” Eur. Phys. J. C **73**, no. 12, 2644 (2013) [arXiv:1304.2159 [hep-th]].
- [14] L. Bonora and D. D. Tolla, “Comments on lump solutions in SFT,” arXiv:1412.0936 [hep-th].